MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 17/1996

Hyperbolic Systems of Conservation Laws

April 28 – May 4, 1996

The conference on hyperbolic systems of conservations laws was attended by 46 participants from 11 countries.

The main idea of this meeting was to bring together people from both the theoretical (22 participants) and numerical (24 participants) aspects of this topic, since the exchange of ideas between these groups is of great importance for further progress in this field. This has been confirmed by many results obtained in the past.

Some month before the conference all participants were asked if they prefered a 40–minute–talk, a 10–minute–talk or to lead a discussion group. Most people indicated that 10 minutes was too short, but many were happy to give a 20–minute–talk. On the basis of the response we scheduled seventeen 40–minute–talks and four 20–minute–talks for the morning sessions, twenty–one 20–minute–talks for the afternoon session and three discussion groups about the following topics:

- Theoretical aspects of multidimensional conservation laws
- How useful are truly multidimensional schemes
- Initial boundary value problems and boundary layers.

It turned out that this combination of short and more comprehensive contributions and the discussion groups was very effective and stimulated a lot of informal discussions during the "free" time.

The main subjects which have been lectured on and discussed during this conference were:

- Numerical schemes in multidimensions
- Improved higher order–schemes in 1–D
- A–priori error estimates
- Existence and uniqueness of solutions for special systems
- Existence of smooth solutions in multidimensions

- Travelling wave solutions
- Perturbations of shock waves
- Conservation laws with relaxation and stiff source terms
- Stability of linear and nonlinear waves
- Applications (liquid–gas flow, astrophysics).

The idea of Professor Kreck to organize the award ceremony of the prize of the Mathematisches Forschungsinstitut Oberwolfach on Wednesday afternoon (instead of the usual walk) was accepted by most of the participants and only a few took a walk and then returned to the second part of the award ceremony. The awards talks were excellent and fit well with the topics of our conference, so this was really a positive addition to the conference.

Finally we would like to thank the Oberwolfach Institute for the excellent meals and lodging which made it possible to work in a stimulating atmosphere.

List of abstracts:

H. D. Alber

Inelastic Material Behaviour of Metals. Transformation of Internal Variables.
Let $\Omega \subset \mathbb{R}^3$ be an open set representing the mass points of a solid body, let $u(x,t) \in \mathbb{R}^3$ denote the displacements, $T(x,t)$ the Cauchy stress tensor and $\rho = \text{constant} > 0$ the density. The initial–boundary value problem governing small deformations of the body consists of the equations

$$\begin{aligned} \rho u_{tt} &= \text{div}_x T \\ T &= D(\epsilon - Bz) \\ z_t &= f(\epsilon, z), \end{aligned}$$

and appropriate initial– and boundary conditions. Here ϵ is the symmetric part of the deformation gradient $\nabla_x u(x,t)$, $z \in \mathbb{R}^N$ is the vector of internal variables, and $\epsilon_p = Bz$ is the plastic strain, B is a linear mapping. f is a given function, which must satisfy the following condition. There exists a function $(\epsilon, z) \mapsto \psi(\epsilon, z) \in [0, \infty)$ such that

$$\begin{aligned} \rho \nabla_\epsilon \psi(\epsilon, z) &= T \\ \rho \nabla_z \psi(\epsilon, z) \cdot f &\leq 0. \end{aligned}$$

ψ is called free energy. For this system of equations an existence theory is sketched, which is based on introducing new internal variables $h(x,t) = H(z(x,t))$, where the vector field $H : \mathbb{R}^N \to \mathbb{R}^N$ is chosen such that the transformed system has a form for which existence can be proved using the theory of evolution equations to monotone operators.

Alberto Bressan

The Semigroup Approach to Systems of Conservation Laws

Every $n \times n$ strictly hyperbolic system of conservation laws admits a Standard Riemann Semigroup. In other words, there exists a globally Lipschitz semigroup, whose domain contains all functions with suitably small total variation, which is compatible with the standard solutions of the Riemann problems.
The talk will outline the main steps in the construction of the semigroup, including sharp decay estimates for positive waves and structural stability results for solutions generated by wave–front tracking approximations.

Gui–Qiang Chen

Global Entropy Solutions with Geometrical Structure for the Inviscid Compressible Flows in Several Space Variables

We are concerned with global entropy solutions for inviscid compressible flows with geometrical structure. Recent developments in this direction are reviewed and the role of the shock capturing methods is discussed. Some efficient shock capturing schemes and their convergence are analyzed in order to compute the corresponding compressible flows and to construct correct approximate solutions.

Bernardo Cockburn

A Priori Error Estimates for Scalar Conservation Laws

A general theory of *a priori* error estimates for scalar conservation laws is constructed by suitably modifying the original Kuznetsov approximation theory.
As a first application of this general technique, we show that error estimates for conservation laws can be obtained without having to use explicitly *any* regularity properties of the approximate solution. Thus, we obtain optimal error estimates for the Engquist–Osher scheme without using the fact (i) that the solution is uniformly bounded, (ii) that the scheme is total variation diminishing, and (iii) that the discrete semigroup associated with the scheme has the L^1–contraction property, which guarantees an upper bound for the modulus of continuity in time of the approximate solution.
As a second application, we focus on how the lack of consistency introduced by the nonuniformity of the grids influences the convergence of flux–splitting monotone schemes to the entropy solution. We obtain the optimal rate of convergence of $(\Delta x)^{1/2}$ in $L^\infty(L^1)$ for consistent schemes in arbitrary grids *without* the use of any regularity property of the approximate solution. We then extend this result to less consistent schemes, called p–consistent schemes, and prove that they converge to the entropy solution with the rate of $(\Delta x)^{\min\{1/2,p\}}$ in $L^\infty(L^1)$; again, no regularity property of the approximate solution is used. Finally, we propose a new explanation of the fact that even inconsistent schemes converge with the rate of $(\Delta x)^{1/2}$ in $L^\infty(L^1)$. We show that this well–known *supraconvergence* phenomenon takes place because the *consistency* of the numerical flux and the fact that the scheme is written in *conservation form* allows the regularity properties of its approximate solution (total variation boundedness) to compensate for its lack of consistency; the nonlinear nature of the problem does not play any role in this mechanism. All the above

results hold in the multidimensional case, provided the grids are Cartesian products of one–dimensional nonuniform grids.

Björn Engquist

Multiphase Geometrical Optics

A system of nonlinear conservation laws for the high frequency approximation of the wave equation is derived. The derivation is based on a kinetic formulation. A closure assumption assuming a finite number of phases at each point in space and home is essential. Numerical solutions are given showing the importance of non split approximation schemes.

Michael Fey

Numerical Methods for Nonlinear Multi–Dimensional Systems of Conservation Laws

The topic of the talk is related to the numerical solution of multi–dimensional systems of conservation laws. It is the aim to obtain methods of high accuracy that show no or only weak dependencies on the underlying grid. This means that the shape of strong gradients or shocks does not change whether they are aligned or oblique to one of the coordinate axes.

We will explain the idea of the method by means of the scalar advection equation. For a finite volume discretization and with the idea of characteristics, it is possible to simulate an advection or transport process with the numerical method that uses the "physical" propagation direction instead of the coordinate axes. This allows to generate high order methods with very compact stencils. In the linear case, the numerical method computes almost the analytic solution.

In the system case, the single propagation direction of the scalar equation has to be replaced by the union of all the characteristic hyper–surfaces. This introduces infinitely many propagation directions into the numerical method. This process can generally be described in the framework of wave propagation phenomena. Since the update to the new time level uses contributions from domains to domains rather than a flux across cell boundaries, the one–dimensional definition of a consistent flux has no analogon in the multi–dimensional case. We will introduce sufficient conditions on the general wave structure that guarantees a consistent flux. Various modifications are possible to obtain certain properties of the numerical method, e.g. smooth "fluxes", simple evaluation etc., and they can easily be checked with these relations. One of the modifications can be interpreted as a local decomposition in time of the non–linear system into a number of scalar advection equations. This allows a rigorous error analysis of the approximation error and allows the extension to high order of accuracy. Thus, we can return to the first problem mentioned of solving scalar equations.

Application of this idea to various systems will be presented, e.g. wave equations, Euler equations and some numerical results are shown.

Heinrich Freistühler

Asymptotic Stability of Traveling Shock Waves

We prove the asymptotic stability of traveling shock waves in arbitrary scalar viscous conservation laws, for arbitrary L^1 perturbations. The flux may be non–convex, non–concave; the shock wave may be characteristic on one or either side; the perturbation may be large. The proof is done in two steps: One first establishes stability for perturbations with small L^1 norm. Then stability for arbitrarily large perturbations, is inferred from the observation that the basin of attraction is open and closed in L^1. (Joint work with D. Serre.)

Marguerite Gisclon

Boundary Conditions for a Strictly Hyperbolic System *via* the Godunov Scheme

We study hyperbolic systems of conservation laws in one space variable, in particular the behaviour of the boundary conditions for the Godunov scheme as the space step tends to zero.

Thanks to entropy estimates, we prove the convergence of the solution of the scheme towards the solution of a hyperbolic initial boundary value problem in the slab $\mathbb{R}^+ \times (0, S)$ where S is less than the shock breaking time.

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James M. Greenberg

Integrating of Hyperbolic Equations with Source Terms

We consider a quasi linear conservation law with an even convex flux function and a given bounded piecewise smooth source term depending on the space variable. We first characterize the solution of the Riemann problem through a new LAX type formula. Then we prove that the associated semi group is a L^1 contraction and obtain an existence theorem for weak solutions. We conclude by constructing Godunov type difference schemes and prove that these schemes are L^∞ stable and preserve stable steady solutions.

Florence Hubert

Fast–Slow Dynamics for Perturbations of Conservation Laws

We study the "large time" behaviour of parabolic perturbation of hyperbolic systems. By "large time", we mean time inversely proportional to the size of the perturbation. We restrict our attention to systems endowed with both linearly degenerate and genuinely nonlinear fields, with periodic initial data.

For "moderate time", the perturbation may often be ignored. For "large time", we observe easily in the scalar case that the linear wave does propagate in a way which depends deeply on the perturbation, whereas the nonlinear one are damped. For systems, these behaviours both occur, but their coupling makes the description harder. We give a qualitative description of the dynamics for "large times", by means of an asymptotic expansion : at the leading level, the nonlinear modes depend only on slow time, whereas the linear mode is a wave whose speed and profile depend also on slow time.

The justification of such an asymptotic expansion is of great importance and far from trivial. We give in this talk a solution to this problem in several cases.

Shi Jin

The Relaxation Scheme for Systems of Conservation Laws

We present the relaxation schemes for systems of conservations laws in several space dimensions. The idea is to use a local relaxation approximation. We construct a linear hyperbolic system with a stiff lower order term that approximates the original system with a small dissipative correction. The new system can be solved by underresolved stable numerical discretizations without using either Riemann solvers spatially or a nonlinear system of algebraic Equations temporally.

Smadar Karni

Computations of Slowly Moving Shocks – An Explanation?

Shock capturing schemes for computations of slowly moving shock fronts may generate disturbances propagating along the characteristics that are not associated with the shock family. These disturbances take the form of a continuous oscillation, and appear as a wavy tail attached to the shock front. They are generated already by first order schemes, but become more pronounced in higher order schemes due to their lower dissipation. A key observation is that the numerical viscosity in the shock family becomes very small for slow shocks, and third order effects are no longer negligible. We have studied perturbations to travelling wave solutions by keeping the 3rd order terms in the modified equations. The perturbations are shown to be generated by a source at the shock 'layer', moving with the shock speed. The disturbances are then projected onto the shock characteristic as well as 'the other' characteristic through a nonlinear coupling term. These disturbances may be interpreted in the context of stability theory of viscous shock profiles.

Barbara Keyfitz

I would like to outline a new approach which Suncica Canic and I have made to studying self–similar two–dimensional problems. There is a supersonic/subsonic dichotomy for unsteady self–similar two–dimensional problems, which makes the mathematical issues similar to those in steady transonic flow. We have found some clues to nonlinear wave interactions by analysing the subsonic region, where a very interesting degenerate elliptic equation governs the flow. The solutions may display singular behavior near the sonic line, and this may explain some of the difficulties in extending mathematical analysis to more than one space dimension, as well as eventually suggesting a resolution of these difficulties.

Christian Klingenberg

The Relaxation Limit for Systems of Broadwell Type

This is joint work with Y.G. Lu. We consider the Cauchy problem for the following system coming from an equation of Broadwell's type:

$$\left.\begin{aligned} \rho_t + m_x &= 0 \\ m_t + (\rho - 4s)_x &= 0 \\ s_t + \frac{\bar{F}(\rho, m, s)}{\tau} &= 0 \end{aligned}\right\} \qquad (0.1)$$

with bounded L^2 measurable initial data

$$(\rho, m, s)_{|t=0} = (\rho_0(x), m_0(x), s_0(x)) \quad . \qquad (0.2)$$

With certain assumptions on (0.1) we show that the solution of the equilibrium system

$$\left.\begin{aligned} \rho_t + m_x &= 0 \\ m_t + (\rho - 4h(\rho))_x &= 0 \end{aligned}\right\} \qquad (0.3)$$

is given by the limit of the solutions of the viscous approximation

$$\left.\begin{aligned} \rho_t + m_x &= \epsilon \rho_{xx} \\ m_t + (\rho - 4s)_x &= \epsilon m_{xx} \\ s_t + \frac{\bar{F}(\rho, m, s)}{\tau} &= \epsilon s_{xx} \end{aligned}\right\} \qquad (0.4)$$

as ϵ and τ go to zero.

Dietmar Kröner

Measure Valued Solutions of Conservation Laws

The question of global solutions in time for general systems of consevation laws is still an open problem. In a recent joint paper with W. Zajaczkowski we have shown at least the existence of a measure valued solution for the Euler equations of gas dynamics, where the conservation–of–energy–equation is replaced by the transport equation for the entropy. We have used an elliptic operator of sixth order in the momentum equation as an ε–regularization. The existence of a solution of the regulaized problem has been obtained by a Galerkin method. Stability estimates for the Galerkin solution hold uniformly and could be carried over to the solution of the ε–regularization uniformly in ε. These estimates imply the existence of a measure valued solution in the limit $\varepsilon \to 0$. It is still not clear how to show that the obtained measure valued solution is a Dirac measure, which would imply the existence of a weak solution.

Jan Olav Langseth

A Three–Dimensional Wave–Propagation Method
for Systems of Conservation Laws

A class of wave propagation algorithms for three–dimensional conservation laws are developed. This unsplit finite volume method is based on solving one–dimensional Riemann problems at the cell interfaces and applying flux–limiter functions to suppress oscillations arising from second derivative terms. Waves emanating from the Riemann problem are further split by solving Riemann problems in the transverse direction to model cross–derivative terms. Due to proper upwinding, the method is stable for Courant numbers up to one. Several examples using the Euler equations are included.

Philippe LeFloch

Non–Classical Shock Waves in Scalar Conservation Laws

In this lecture, I report on a joint work with Brian T. Hayes, University of Southern California, Los Angeles. We study the non–classical shock waves that arise as limits of certain diffusive–dispersive approximations to hyperbolic conservation laws. Such shocks are associated with non–convex flux–functions and connect regions of different convexity. They are undercompressive, have negative entropy dissipation for one entropy function, but do not obey the classical Oleinik entropy inequalities.

We shall discuss the scalar conservation law

$$\partial_t u + \partial_x f(u) = 0, \qquad u(x,t) \in \mathbb{R},\, x \in \mathbb{R},\, t > 0,$$

where the flux–function $f : \mathbb{R} \to \mathbb{R}$ is non–convex. We mostly focus on the prototypical case of the cubic flux $f(u) = u^3$. We give necessary conditions for the existence of non–classical shock waves, and construct them as limits of traveling wave solutions for several diffusive–dispersive approximations.

We introduce a “kinetic relation” which acts as a selection principle to pick up a unique non–classical solution to the Riemann problem. The convergence to non–classical weak solutions for the Cauchy problem is investigated. Finally, using numerical experiments, we demonstrate that the Beam–Warming scheme produces non–classical shocks while no such shocks are observed with the Lax–Wendroff scheme. All of these results depend crucially on the sign of the dispersion coefficient.

Yun–Guang Lu

The Study on the Degenerate Hyperbolic System and on the Degenerate Parabolic Equation

This talk consists of two parts. In part one, we introduce a method to use the compensated compactness with the entropy–entropy flux pairs of Lax type to prove the convergence of the approximate solutions (viscosity solutions, Lax–Fridrichs numerical solutions or Godunov numerical solutions) for some nonstrictly hyperbolic systems, such as LeRoux system; one–dimensional compressible fluid flow; 2×2 chromatography system; a special 2×2 system of conservation laws with quadratic flux and so on. In part two, we consider the Cauchy problem

$$\begin{cases} u_t + f(u)_x = G(u)_{xx} \\ u(x,0) = u_0(x). \end{cases} \tag{0.5}$$

We give a critical condition on the nonlinear function G as follows: Let

$$Z = meas\{u : G'(u) = 0\}. \tag{0.6}$$

If $G'(u) \geq 0$ and $Z = 0$, then the Cauchy problem (0.5) has a unique Hölder continuous solution. If the set Z is positive, then the solution of the Cauchy problem (0.5), in general, is discontinuous. In the second part, we also introduce a method to obtain the Hölder continuous solution with explicit Hölder exponent for the high dimensional Porous Media equation

$$u_t = \Delta u^m \tag{0.7}$$

The second part in this talk is joint work with W. Jäger.

Pierangelo Marcati

Singular Limits for Hyperbolic Systems of Conservation Laws

We expose a general theory for analyzing the singular behaviour of hyperbolic systems under non–hyperbolic scaling.

K.W. Morton

Evolution–Galerkin and Generalised Godunov Schemes for Multidimensions

For a scalar problem in 1D there is a wide consensus on the key elements of an effective scheme: use the characteristics; p.w. constant or p.w. linear basic approximation; recovery by continuous parabolics to give third order accuracy; solution adaptivity to deal with shocks. Either of the main formulations, evolution Galerkin or Godunov extend readily to conservation law systems. Even for the scalar problem there is less agreement in multidimensions. However, use of the characteristics in either formulation gives the crucial terms, while other formulations do not always do so. The real test is multi–D systems, typified by the second order wave equation and the Euler equations. For the former, an exact evolution operator can be given in terms of integrals along bicharacteristics and over the characteristic cone; and this has been used as the basis for a number of numerical schemes. Ostkamp has also shown how it can be applied to the Euler equations.

Ingo Müller

The Symmetric Hyperbolic Systems of Extended Thermodynamics
– Light Scattering and Shock Wave Structure

The symmetric hyperbolic systems of extended thermodynamics result from the kinetic theory of gases by forming moments. Well–known is the Grad 13–moment theory but this offers no great improvement over the Navier Stokes theory, even though it is hyperbolic. Many moments are required for a better description of nature when rapid changes and steep gradients are involved. This proposition is illustrated for light scattering and shock wave structures.

Claus–Dieter Munz

Numerical Methods for Low Mach Number Flow

An asymptotic analysis of the compressible Euler equations in the limit of vanishing Mach numbers is used as a guideline for the development of numerical schemes in the weakly compressible regime. Multiple pressure variables are introduced which accounts for the three physically distinct roles of the pressure as a thermodynamic variable, an acoustic wave amplitude, and the balancing agent for inertial forces. Advection of mass and momentum as well as long wave acoustics are discretized explicitly, while in solving the sonic terms, the scheme uses an implicit pressure correction formulation to guarantee both divergence–free flow in the zero Mach number limit and appropriate representation of weakly nonlinear acoustic effects for small but finite Mach numbers. This asymptotics based approach may be used to extend compressible as well as incompressible flow solvers to the weakly compressible regimes.

Benoit Perthame

A Numerical Approach for Two Phases Flows

The engineering literature proposes models for two phases flows of the following type:

$$\begin{aligned} \partial_t \alpha_k \rho_k + \partial_x(\alpha_k \rho_k u_k) &= 0 \\ \partial_t \alpha_k \rho_k u_k + \partial_x(\alpha_k \rho_k u_k^2 + \alpha_k p) &= p\partial_x \alpha_k + \text{added mass forces} \\ \partial_t \alpha_k E_k + \partial_x \alpha_k (E_k + p) u_k &= -p\partial_t \alpha_k. \end{aligned}$$

The exact form of the added mass forces (and other terms we do not mention here) depends on the flow. It is therefore useful to propose methods which do not rely on this specific form although these terms enforce the hyperbolicity of the system. We propose such a method which is based on classical Riemann solvers for classical hydrodynamics. (Common work with F. Coquel, K. El Amine, E. Godlewski, P. Rascle).

Christian Rohde

Weakly Coupled Systems of Conservation Laws

We consider a nonlinear system of hyperbolic conservation laws in several space dimensions where the coupling of the equations is only due to the source functions. Problems that lead to these weakly coupled systems can be found for example in combustion theory, hydrology or mathematical biology.

Based on the well–known viscosity method unique existence of appropriately defined entropy solutions is derived if an associated parbolically regularized problem admits bounded classical solutions. Some physically relevant model problems are proven to have an entropy solution.

Concerning the numerical solution of weakly coupled systems we present a finite volume method on unstructured grids. Extending a result of DiPerna we can give criteria that ensure the convergence of the finite volume method for initial value problems. By application of this theorem strong convergence to the exact entropy solution is verified.

Mirko Rokyta

A–Priori Error Estimates for Upwind Finite Volume Schemes for Convection Dominated Diffusion Equa

We consider the following boundary value problem

$$Lv := -\varepsilon\Delta v + \mathrm{div}(bv) + cv = f \quad \text{in } \Omega\,, \qquad (0.8)$$

$$v = 0 \quad \text{on } \partial\Omega\,, \qquad (0.9)$$

where Ω is a convex polygonal domain in $\mathbb{R}^2$ and $b(x)$, $c(x)$, $f(x)$ are functions which are smooth enough such that $0 < c_0 \le c(x) \le c_1$, $\mathrm{div}\, b = 0$.

We use upwind finite volume discrete operator to obtain approximate solution to the problem. Under suitable conditions on smoothness of the exact solution and the function f we first show that, if the discrete operator is formally of first order, the a–priori error estimate is of order $O(h)$. Then we prove that, if a second order linear reconstruction operator is used for discretizing the diffusion term, the a–priori error estimate is of order $O(h^2)$ in subdomains not containing local extrema of the solution. (Joint work with D. Kröner and M.Wierse.)

Tommaso Ruggeri

Hyperbolic Principal Subsystems:
Entropy Convexity and Subcharacteristic Conditions

We consider a system of N balance laws compatible with an entropy principle and convex entropy density. Using the special symmetric form induced by the "main field", we define the concept of "principal subsystem" associated to the system. We prove that the $2^N - 2$ principal subsystems are also symmetric hyperbolic and satisfy a

”subentropy law”. Moreover we can verify that for each principal subsystem the maximum (minimum) characteristic velocity is not larger (smaller) than the maximum (minimum) characteristic velocity of the full system (subcharacteristic conditions). We present some simple examples in the case of the Euler fluid. Then in the case of dissipative hyperbolic systems we consider an equilibrium principal subsystem and discuss the consequences in Extended Thermodynamics. Finally, in the moments approach to the Boltzmann equation we prove, as a consequence of the previous result, that the maximum characteristic velocity evaluated at the equilibrium state does not decrease when the number of moments increases.

Denis Serre

Global Smooth Multidimensional Compressible Flows with a Finite Mass
One proves the existence of non–trivial such flows for a polytropic gas. The pressure law is either $p = \rho^\gamma$ (isentropic case) or $p = (\gamma - 1)\rho e$ (non isentropic), with $\gamma \leq 1 + 2/d$, d being the dimension of the physical space. The monoatomic gas corresponds to $\gamma = 1 + 2/d$; in that special case, the global existence follows from a local existence and a ”conformal invariance” of Euler’s equations.

Michael Shearer

Shear Band Formation in Granular Materials
The appearance of shear bands in an elastoplastic solid is associated with a change of type in the pde’s: Loss of hyperbolicity in dynamic equations, loss of ellipticity in quasi–static equations. Here, the evolution of the deformation in simple shear is followed up to the time of shear band formation. As the shear band forms, it does so at a weak spot in the material; in the analysis, we introduce a corresponding small nonuniformity in material properties. We find that the deformation is extremely nonuniform in a small interval of time before the shear band appears.

Chi–Wang Shu

Weighted Essentially Non–Oscillatory Schemes
Weighted ENO (Essentially Non–Oscillatory), or WENO, schemes are based on ENO schemes, to use adaptive stencil idea to design high order shock capturing schemes. Instead of using one of the candidate stencils as in ENO, WENO uses a linear combination of all candidate stencils. We will discuss ways to choose the nonlinear weights with the objective of getting a smooth, high order stencil in smooth regions and sharp, monotone shock transition.

David Sidilkover

New Discretizations and Fast Solvers for the Inviscid Flow Equations
One of the most glorious pages in the history of numerical analysis was the construction of the so–called high–resolution schemes for gas dynamics. These schemes had to incorporate a certain smoothness monitor (i.e. to be highly nonlinear) in order to combine second–order accuracy and non–oscillatory properties. The drawback of

this approach is that high–frequency error components are not sufficiently visible to the residuals of such schemes. Therefore, it may be inherently impossible to construct a relaxation with good smoothing properties using the discretizations of this type. This explains in part a poor performance of the existing multigrid flow solvers. This difficulty can be traced to the particular way the high–resolution mechanism (nonlinearity) is incorporated within the schemes (i.e. to the dimensional splitting). We shall present briefly a genuinely multidimensional approach towards the construction of high–resolution schemes. The key advantage of this approach is that it allows to achieve non–oscillatory properties of the discrete scheme without damaging its stability. This facilitates the possibility to construct a simple (namely, Gauss–Seidel) relaxation with good smoothing properties for the entire range of flow regimes.
The resulting genuinely multidimensional scheme has intriguing links to some standard schemes used for incompressible flow computations. Exploration of these links facilitated in part a construction of the discretization suitable for the incompressible flow computations. This scheme is formulated on (non–staggered) triangular unstructured grids. It allows to achieve optimal multigrid efficiency (i.e. efficiency identical to that for Poisson equation). since it separates the elliptic and advection factors of the system. The most remarkable property of this scheme is its extreme *simplicity* together with the simplicity of the entire solver:

– it relies on the primitive variables;

– simple treatment of the boundary conditions;

– use of the Collective Gauss–Seidel relaxation.

This approach readily extends to the compressible subsonic flow case. It faciliates the optimal efficiency of the multigrid solver for compressible case as well (i.e. the same efficiency that can be obtained when solving a Full–Potential equation). The important point here, besides the simplicity of this approach, is that there is no loss of accuracy/efficiency in the incompressible limit. The scheme/algorithm constructed for the compressible flow computations becomes identical to those used for the incompressible flow mentioned above.
Numerical experiments illustrating the efficiency and accuracy of these new approaches will be presented.

Carlo Sinestrari

Travelling Waves for Conservation Laws with Source

We investigate the qualitative properties of the solutions of the scalar conservation law with source

$$\partial_t u(x,t) + \partial_x f(u(x,t)) = g(u(x,t)) \qquad x \in \mathbb{R}, t \geq 0$$

under various types of initial data (periodic, with compact support, or Riemann–like).
We show that the asymptotic profile of the solutions is given by a superposition of shock waves and travelling waves, instead of the rarefaction waves familiar from the

$g = 0$ case. The travelling waves are of two types: there are continuous waves connecting two consecutive zeros of g and discontinuous one oscillating around a zero of g. We prove that waves of the latter form have an unstable character, and that for a generic class of initial data the asymptotic profile of the solution does not contain any of such waves. We also analyze how the rate of convergence of the solutions to their asymptotic profile is affected by the presence of the source.

Marshall Slemrod

Relaxation Limit for Non–Ideal Gases

The shock structure of materials exhibiting phase transitions has been a popular subject recently. This talk will discuss recent investigations into the shock structure and viscous compressible gas dynamics of a condensing vapor. The main tool are a discrete velocity Boltzmann like model and some recent ideas of O. Penrose on metastability.

Thomas Sonar

Recent Developments in the Construction of ENO Schemes on Unstructured Meshes

Essentially non–oscillatory recovery is the most expensive algorithmic part in finite volume approximations of hyperbolic conservation laws. We describe the use of generalised Mühlbach expansions in order to allow a cheap construction of the recovery function. Generalised multiresolution algorithms to further increase the efficiency of these schemes are discussed. These methods work on arbitrary domains and are independent of the underlying grid partitioning.

P. Souganidis

Existence and Stability of Entropy Solutions for the Hyperbolic System of Isentropic Dynamics

In this talk I will discuss the existence and compactness (stability) of entropy solutions for the hyperbolic systems of conservation laws corresponding to the isentropic gas dynamics, where the pressure and density are related by a γ–law, for any $\gamma 1$. Our results (joint work with P.–L. Lions and B. Perthame) considerably extend and simplify the program initiated by DiPerna and provide a complete existence proof. Our methods are based on the compensated compactness and the kinetic formulation of systems of conservation laws.

Anders Szepessy

Multigrid Methods for Shocks

In the talk I present two theorems on the residual damping in multigrid methods solving convection dominated diffusion equations and shock wave problems, discretized by the streamline diffusion finite element method. One theorem shows that a V–cycle, including sufficiently many pre– and post–smoothing steps, damps the residual in L^1_{loc} for a constant coefficient problem with small diffusion in two space dimensions, without the assumption that the coarse grid is sufficiently fine. The second theorem prooves a similiar result for a certain continuous version of the two grid method, with isotropic artificial diffusion, applied to a two dimensional Burgers shock wave problem.

Eitan Tadmor

Nonoscillatory Central Schemes for Nonlinear Conservation Laws and Related Equations

During the recent decade there was an enormous amount of activity related to the construction and analysis of modern algorithms for the approximate solution of nonlinear hyperbolic conservation laws and related problems. To present the successful achievements of this activity, we review some of the analytical tools which are used in the development of the convergence theories associated with finite–difference, finite–element, finite–volume and spectral approximations of nonlinear hyperbolic systems. In particular, we focus our discussion on modern high–resolution finite–difference approximations to hyperbolic conservation laws.

We highlight those schemes which are based on *central* differencing: their building block is the use of staggered grids. The main advantage is *simplicity*, since no Riemann problems are involved. In particular, we avoid the time–consuming field–by–field decompositions required by (approximate) Riemann solvers of upwind difference schemes. Excessive numerical dissipation is compensated by using modern high–resolution, non–oscillatory reconstructions. Moreover, in the scalar case we prove total–variation bounds, one–sided Lipschitz bounds (– which in turn yield precise error estimates), as well as entropy stability estimates.

Finally, a variety of numerical experiments – including second– and third–order approximations to Euler and MHD equations, demonstrate that these central schemes offer *simple, robust, Riemann–solver–free* approximations for the approximate solution of one– and two–dimensional systems. At the same time, these central schemes achieve the same quality results as the high–resolution upwind schemes.

Athanasios Tzavaras

Self–Similar Viscous Limits and the Riemann Problem for Nonlinear Hyperbolic Systems

We consider the problem of constructing solutions $u(x/t)$, $\xi = x/t$ of the Riemann problem for an $N \times N$ system of strictly hyperbolic conservation laws for data $|u_+ - u_-| << 1$ via solving

$$\begin{aligned} -\xi u_\epsilon' + F(u_\epsilon)' &= \epsilon u_\epsilon'' \\ u_\epsilon(\pm\infty) &= u_\pm . \end{aligned}$$

It is shown that u_ϵ is of uniformly bold variation and thus $u_{\epsilon_n}(\xi) \to u(\xi)$ and that $u(x/t)$ solves the Riemann problem. The solution satisfies the Lax shock conditions and at shocks u_ϵ has the internal structure of traveling waves. The solution of the Riemann problem in the case of nonconservative hyperbolic systems (joint work with Ph. LeFloch) is also discussed.

Roland Vilsmeier

Flow Computations Employing Explicit Adaptive Methods on 3D Hybrid Grids
Aim of the investigations is to create reliable and efficient algorithms to solve the Euler and Navier–Stokes equations in geometrically complex geometries. Due to the very bad properties of tetrahedral elements when discretizing terms containing second spatial derivatives, a hybrid grid approach, employing automatic mesh generators will be prefered in future.
As a first step in this direction a numerical method based on finite volumes is proposed, that, due to its simple data structure, is able to be employed on any grid. At present, solutions are obtained on fully unstructured, tetrahedral grids as well as manually arranged grids employing different element types in azonal approach. These grids contain prismatic layers and cartesian blocks covered by pyramids, while the empty spaces are "filled" by tetrahedra.
Integration is carried out explicitly. If local stability conditions vary essentially throughout the computational domain, a renewed version of a previously developed multisequence Runge–Kutta method, now fully conservative, can be employed.

Zhouping Xin

On Initial–Boundary Value Problems and Boundary Layer
Behaviour of Weak Solutions for Conservation Laws
We present some results on boundary conditions formulations for systems of hyperbolic conservation laws and study the traces of not only entropy–fluxes but also density variables. We apply this theory to some nonstrictly hyperbolic systems and scalar convex conservation laws. The solutions under study are generated either by viscous approximations or Godunov schemes. Some interesting phenomena are discussed.

Robin Young

Growth Rates for 3×3 Conservation Laws
We consider the Cauchy problem for 3×3 conservation laws in one space dimension, including Euler's equations for gas dynamics. We consider initial data having large total variation and small oscillation. We briefly show some examples which demonstrate that the total variation is not bounded in general. By considering the scaling properties of solutions, we identify a new length scale which determines the growth rate of the total variation. We then restrict to systems with a Riemann coordinate, which includes the Euler equations. We prove a large–time existence theorem and show that the total variation grows at most exponentially with growth rate determined by the length scale, which is obtained from the initial data. Most of this is joint work with Blake Temple.

Yanni Zeng

Conservation Laws of Composite Type

We study systems of conservation laws which contain both hyperbolicity and dissipativeness. Large time behaviour is studied for solutions that are small perturbations of a constant state. We give pointwise estimates, both in space and in time, on the solution for large time for Navier–Stokes equations without heat conductivity.

Guohui Zhou

Accretion Disk Problems

Accretion disk problems arise in astrophysics if a star is rotating in a high speed and burning out in the stage of expansion. The matter flows from the star with supersonic speed and is accreted near the star. This accreted body revolves on its own center. With the increase of the angular speed, an accretion disk is formed. Mathematically, the movement of the accretion disk fulfills the nonstationary Navier–Stokes equations in cylindrical coordinates.

With the operator splitting method, we decouple the system in single equations, so that we solve scalar equations in each time step. Each decoupled equation is convection–dominated. To maintain the stability and to get sharp front of the shocks which occur near the inner boundary, we use the streamline diffusion finite element method to solve each equation. Shock–capturing techniques are also used to damp away the possible over– and undershootings. Test computations for model problems in one dimension are presented.

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